

Kant's Logic

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Abstract

This book develops a formal and philosophical reconstruction of Kant's general and transcendental logic. Its aim is to understand the details of Kant's specific logical commitments—concerning, for instance, the possible forms of judgment and the traditional square of opposition with conversion—as well as how these relate to certain more general or high-level philosophical commitments, such as his conception of the understanding as a capacity for rule-governed activity. The project is motivated by the thought that these different levels of analysis can illuminate one another, and that modern tools can aid in the investigation. An account that validates Kant's preferred logical laws but treats judgment as an attitude to a static proposition misses his conception of judgment as an activity through which truth-evaluable content is first constituted. An account that emphasizes activity and synthesis without explaining why the square of opposition, conversion rules, and syllogistic figures have the structures Kant assigns them remains underdetermined. We therefore proceed in both directions: from the elementary logical forms and inferences toward an act-theoretic interpretation of judgment, and from that interpretation back toward a more adequate understanding of what the formal constructions represent.

Synopsis

The reconstruction proceeds in stages. An initial fragment of general logic contains only universal and particular quantity, affirmative and negative quality, and categorical relation, with quantity ranging over possible determinations of concepts rather than over existing or possible objects. A distinctive, anti-rationalist principle of Kant's logic is that there is no bottom to conceptual determination. We show how this principle allows us to derive the full traditional square of opposition with conversion, without the need to appeal to existing or possible objects. We also derive the categorical syllogistic and explain Kant's view of the privileged place of the first syllogistic figure. The initial fragment is then extended with hypothetical relation and modality to complete general logic. At this stage the static semantics is reinterpreted dynamically: unmodalized formulas specify rules for possible syntheses, while modality characterizes sheer exercises of those rules. This provides a unified account of three further Kantian commitments: that judgment is an activity, that all judgments involve a modal aspect, and that truth-evaluability first arises at the level of judgment, so that the unmodalized content of judgment cannot itself be truth-evaluable. A second, material extension is then added that assigns concepts their instantiating objects and supports a distinct material truth relation. This allows us to recover a second full square of opposition for material truth, now including accounts of existential import and empty terms, and with suitable bridge principles to the first, logical square. It also allows us to explain Kant's famous remarks regarding existence. Finally, the transcendental expansion of general logic adds singular quantity, infinite quality, disjunctive relation, and apodictic modality. These new moments of judgment are shown to be both interdependent and irreducible to previous moments only under considerations of conceptual content from which general logic abstracts. The completed architecture allows us to appreciate the additional internal structure that content brings to our squares of opposition, which in turn allows us to articulate a novel account of how general logic abstracts from such matters. The result of our reconstruction is a two-extension, act-theoretic logic, broadly within the Aristotelian tradition, but in which judgments are modal actualizations of rules and logical and material truth are systematically coordinated without being reduced to one another.

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1 Introduction

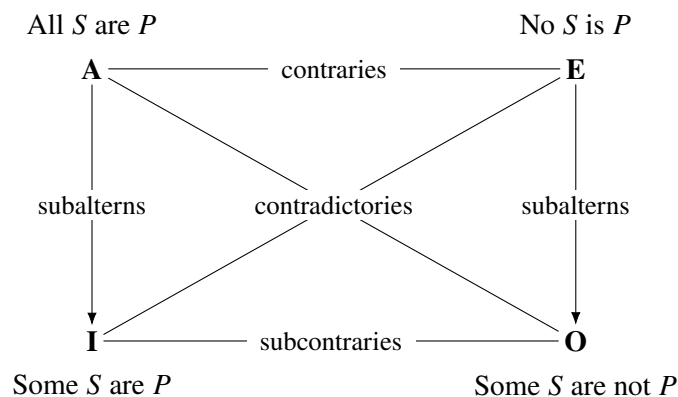
This book develops a formal and philosophical reconstruction of Kant's general and transcendental logic. Its aim is to understand the details of Kant's specific logical commitments—concerning, for instance, the possible forms of judgment and the traditional square of opposition with conversion—as well as how these relate to certain more general or high-level philosophical commitments, such as his conception of the understanding as a capacity for rule-governed activity. The project is motivated by the thought that these different levels of analysis can illuminate one another, and that modern tools can aid in the investigation. An account that validates Kant's preferred logical laws but treats judgment as an attitude to a static proposition misses his conception of judgment as an activity through which truth-evaluable content is first constituted. An account that emphasizes activity and synthesis without explaining why the square of opposition, conversion rules, and syllogistic figures have the structures Kant assigns them remains underdetermined. We therefore proceed in both directions: from the elementary logical forms and inferences toward an act-theoretic interpretation of judgment, and from that interpretation back toward a more adequate understanding of what the formal constructions represent.

1.1 Three puzzles

I begin by outlining three puzzles together with their proposed solutions. This provides some initial indication of how far a reconstruction of Kant's logic may have to depart from familiar modern frameworks. The first puzzle is well known and by no means specific to Kant: it concerns existential import and the traditional square of opposition. The second and third puzzles are more specific to Kant and yet often overlooked: they concern Kant's treatment of modality and his account of the difference between concepts and judgments. Each begins from a relatively local interpretative problem. Taken together, they suggest the general picture that will be developed in this book.

1.1.1 The square of opposition without existential import

The traditional square of opposition organizes four categorical forms and the logical relations among them:



Contradictories can be neither true together nor false together. Contraries cannot both be true, though they may both be false; subcontraries cannot both be false, though they may both be true. Subalternation runs from the universal to the corresponding particular: the truth of *A* requires the truth of *I*, and the truth of *E* requires the truth of *O*; equivalently, the falsity of the particular requires the falsity of its universal superaltern.

Modern logic has little use for the full traditional square. The standard modern translations of the four categorical forms are:

$$\begin{aligned} A : \forall x(Sx \rightarrow Px), \quad E : \forall x(Sx \rightarrow \neg Px), \\ I : \exists x(Sx \wedge Px), \quad O : \exists x(Sx \wedge \neg Px). \end{aligned}$$

If we assume a standard objectual semantics and classical truth-conditions, then we lose most of the relations represented in the square. For suppose that nothing is S . The two universal judgments are then both vacuously true, while the two particular judgments are both false. As a result, only the diagonal relations of contradiction survive: A contradicts O , and E contradicts I . The remaining relations of the traditional square all fail. A and E can both be true and so are no longer contraries; I and O can both be false and so are no longer subcontraries; and neither universal entails its corresponding particular so we lose subalternation. Given the standard modern translations, the full square can be recovered only by restricting attention to interpretations on which something is S :

$$\exists x Sx.$$

This is relevant because Kant retains the traditional square of opposition (*Jäsche Logic* §§ 46–50), yet his general logic does not assume that something is S , at least not if this means that a concept is instantiated by an existing object. General logic cannot assume that any individual object exists at all. For one thing, objectual existence claims are always a posteriori, for Kant (A178/B211, A226/B279); general logic and the square of opposition are a priori (A53/B77). How, then, can Kant retain the traditional square of opposition?

It is not exactly breaking news that the standard modern translations may not be the best way to parse how Kant understands the four categorical forms, and there are various traditional and modern ways to recover the square by appealing to alternative translations and subtle considerations concerning existential import and empty terms; we return to these later. But what is of interest here is that an examination of Kant's discussions of conceptual extension, subordination, and determination reveals another possibility. Quantity may concern not only the objects falling under a concept but also the possible further determinations of a concept. On this interpretation, it turns out, the work performed in objectual semantics by an existential assumption can instead be performed by a structural principle about the conceptual hierarchy: that there is no absolutely infima species, or lowest concept, since every concept admits of possible further determination. This is striking because Kant makes just such a principle central to his logic and it marks a decisive break with his rationalist predecessors (*Jäsche Logic* § 11; cf. A655–57/B683–845). We will show exactly how this principle reinstates the full traditional square, as well as the additional conversion rules that accompany it and which Kant also admits. And we explain how Kant's rationalist predecessors have to appeal to quite different resources to achieve the same result.

The first puzzle points towards a distinction between objectual and conceptual extensions of concepts, and ultimately between material and logical truth. It thereby opens the prospect of two coordinated squares of opposition: a logical square grounded in indefinitely extendible conceptual determination, and a material square grounded in objectual instantiation. Notably, neither requires an ontology of merely possible objects.

1.1.2 Modality, truth-evaluability, and the primacy of judgment

In modern modal logics, modal operators ordinarily attach to contents that are already propositionally complete and truth-evaluable. This holds whether the operators are interpreted alethically, epistemically,

or deontically. And it holds whether they are interpreted intensionally, for example as quantifying over possible worlds. Their interpretation determines how a proposition p is evaluated, but p is already the right kind of content to bear a truth-value before it becomes the complement of a modal operator. Thus the operator qualifies the proposition's modal status rather than constituting it as propositional, or truth-evaluable, in the first place. For example, given a proposition p , $\Box p$ and $\Diamond p$ may qualify that same proposition as necessary or possible, which may in turn be understood in terms of possible worlds—but whatever the details, what falls within the operator's scope is already a complete propositional content.

It is tempting, indeed quite standard, to understand Kant's modalities in just this way. I do not mean that it is standard to understand Kant's logical modalities as alethic, still less as quantifying over possible worlds. Rather what is standard is to understand Kant's logical modalities as operating on this kind of content, i.e. on content that is already or independently truth-evaluable. On this picture, the matter of a judgment—the particular concepts S and P —is first structured by the forms of quantity, quality, and relation to yield a complete truth-evaluable proposition, such as “All S are P .” Modality then attaches to this proposition, to qualify it or its judgment in some way. To modern ears, this can sound like a natural interpretation of Kant's claim that “besides quantity, quality, and relation there is nothing more that constitutes the content of a judgment” (A74/B100): by ‘the content of a judgment’, or what is to be judged, Kant means propositional content, or what is true or false (see e.g. Bader).

However, two central Kantian commitments taken together place serious pressure on any such reading. First, Kant holds that every judgment has a modality—there are no unmodalized or amodal judgements (A70/B95). Second, Kant holds that truth and falsity belong only to judgments, which suggests that truth-evaluability first arises at the level of judgment (A293/B350). It would seem to follow that the pre-judgmental, unmodalized combination of S and P that constitutes what Kant calls ‘the content of a judgment’ cannot yet itself be a truth-evaluable proposition. Instead, modality must attach to something pre-propositional and participate in the transition or activity through which truth-evaluable judgment proper is constituted. Kantian modalities do not appear to operate on truth-evaluable propositional contents.

It bears repeating how general this point is. It applies however we interpret modal operators, broadly construed. For the issue is not—at least not yet—how we interpret the modalities. It is rather how we interpret that to which the modalities attach. Thus it also follows, for instance, that Kant's distinction between the content of judgement and the modal manner in which that content is to be judged does not track a Fregean distinction between force and content, where the content is what can be true or false. Kantian amodal content cannot be true or false.

One option at this point might seem to be the following (see Stephenson): to instead treat the pre-judgemental unmodalised content of judgement as merely a complex concept, awaiting semantic completion, or saturation, to become truth-evaluable. On this view, what quantity, quality, and relation determine is a complex concept formed from S and P , for instance $\langle S \text{ and } P \rangle$ or $\langle S \text{ and not-}P \rangle$. The modal moment of judgement would then assert that this complex concept may be, is, or must be instantiated. On this picture, each modal determination operates after the manner of a Fregean second-level concept, taking a first-level concept as its argument to yield something truth-evaluable. (Only the assertoric case corresponds directly to ordinary existential quantification; the problematic and apodictic cases would be modal analogues of it.) This proposal fits with a number of things Kant says and avoids the previous issue, but ultimately it cannot be satisfactory. For one thing, it would run head long into the next puzzle. As we will see, it turns out that the problem here is how to understand modality as making the transition to truth-evaluability without making *any* distinct semantically identifiable *addition* at all. This, I will argue, is what Kant really means when he says that modality “contributes nothing to the content of the judgment” (A74/B100).

Fortunately there is an alternative route that satisfies this constraint while avoiding the initial problem: an unmodalized formula should be interpreted not as a proposition or as a complex concept but as a *rule* specifying a possible way of unifying conceptual determinations. The particular concepts involved in pre-judgmental content provide the particular conceptual determinations, while the forms of quantity, quality, and relation provide the way in which these determinations are to be unified; they specify the rule's scope, direction, and organization. Modality then concerns the sheer exercise of such rules—the act of determining in accordance with the rule.

1.1.3 What distinguishes a concept from a judgment?

The third puzzle is closely related to the second. We will see later that it arises throughout the core corpus in the intimately related things Kant says about the nature of judgment, the copula, and modality. But it can perhaps be seen most clearly in Kant's correspondence with J. S. Beck (11:338–348). Beck asks how a concept, e.g. *white horse*, differs from the corresponding judgment, *the horse is white*. The same conceptual materials appear in both. What converts the first into the second?

A familiar modern answer distinguishes their semantic types. A concept may be treated as a property, predicate, set, or unsaturated function. A proposition, by contrast, is a complete truth-evaluable unit. The transition from concept to proposition is accordingly explained through the production of a different kind of semantic entity, perhaps by saturating a function with an argument or applying a predicate to an object.

Kant's answer is very different. He contrasts thinking of the concept with thinking of “the activity of my determining” that concept. The expression *the white horse* presents a concept as already determinate with respect to whiteness; *the horse is white* presents the activity of determining. The difference is thus not explained simply by adding a new semantic object to the same conceptual materials. Nor is judgment a further attitude adopted towards, or modal qualification of, a proposition already constituted independently of that attitude or qualification. Kant locates the relevant difference in sheer activity: the transition from a represented determination to the act of so determining.

This answer connects directly with the puzzle about modality. If an unmodalized combination of *S* and *P* specifies a possible way of determining one concept through another, it can be understood as a rule rather than as an already completed judgment or truth-evaluable propositional content. Judgment arises when the understanding modally exercises that rule. In this way, the understanding is a *capacity* for rule-governed activity that is realised by a *power* of (modal) judgment.

There is also a connection to the first puzzle. The logical extension of a concept consists of possible determinations of that concept, which is to say it consists of further concepts that stand as species to the initial genus. The structural claim that there is no lowest species is then what ensures that logical extension is never empty, which in turn is what validates the square of opposition regardless of existential import or an ontology of possibilities. This can all be represented set-theoretically, and that will be useful for deriving the square. But ultimately we should not think of the whole infinite conceptual hierarchy as a pre-existing, self-standing structure. It should instead be understood as a static representation of the open-ended possibilities for acts of conceptual determination in judgment. Logic, for Kant, concerns the “function of the understanding in judgements” (A70/B95)—it concerns what we *do* in judging.

1.1.4 Towards a different framework

The three puzzles together motivate the framework developed in what follows. Concepts will first be treated as possible determinations standing in relations of subordination. Quantity, quality, and relation specify rules governing possible ways of unifying those determinations. These rules can initially be

represented statically, allowing us to examine their formal structure without yet treating the corresponding formulas as fully actualized judgments. With the addition of modality the semantics will then be interpreted dynamically, as concerning the exercise of a rule through which judgment proper—and with it truth and falsity—arises.

This framework does not exclude objects, objectual extension, or material truth. Nor does it deny that objectual semantics supplies one legitimate interpretation of categorical forms. It claims only that this cannot be the whole of Kant's logical framework. We shall distinguish logical extension from objectual extension, and logical truth from material truth, and show how each yields a complete square of opposition, with conversion, on a different basis. The later transcendental expansion will then show how the structure already established by general logic becomes more finely articulated when conceptual content and objective application are taken into account.

1.2 Plan of the book

The division between general and transcendental logic structures the book as a whole. Throughout, the formal reconstruction is given in a modern classical meta-language. The quantifiers, truth-functional connectives, and set-theoretic relations used in its semantic clauses and proofs belong to that meta-language, not to the Kantian object-language.

Part I constructs general logic in stages. Section 2 restricts attention to universal and particular quantity, affirmative and negative quality, and categorical relation. It introduces a strict partial order of proper determination, logical extension, Kant's distinctive No-Bottom Principle, and the four categorical forms. Section 3 then turns from the construction of judgments out of concepts to the construction of inferences out of judgments. It proves the complete logical square of opposition along with the additional conversion rules Kant admits, and it identifies the exact work done by the No-Bottom Principle. It also derives the categorical syllogistic and explains Kant's distinctive view of the privileged place of the first figure.

Section 4 then moves from categorical mediation to hypothetical grounding. Hypothetical syllogism makes the modal status of components, premises, and conclusions explicit and provides initial motivation for the dynamic reinterpretation of bare formulas as rules. Problematic, assertoric, and apodictic modalities characterize ways in which those rules are exercised, while genuine inference is distinguished from unrestricted semantic consequence—the conception of consequence at work in Kant's understanding of hypothetical judgment is a grounding-theoretic relevance-sensitive notion.

Section 5 introduces non-logical, material extension and the quality-sensitive material truth-conditions. It reconstructs a second full square of opposition, compares its basis with that of the logical square, returns to hypothetical judgment with real rather than merely logical grounds, and considers bridge principles between the two extensions. Section 6 uses this framework to distinguish existential positing from real conceptual determination and relates the account to modern semantics and Kant's famous remarks about existence.

Part II turns to transcendental logic. Section 7 introduces singular use as the pointification of a general concept and shows why an arbitrary selected-member interpretation does not preserve syllogistic validity. It then introduces the highest concept and uses it to formulate sheer existence claims, both general and singular. Section 8 distinguishes simple negative from infinite judgment, identifies three non-iterable forms of negation, and reconstructs disjunctive judgment as an exclusive and exhaustive content-based partition. Section 9 brings the expanded opposition structure together with the dynamic modal architecture and returns to apodictic force as the status of conclusions grounded in genuine inference.

A conclusion summarises the key results and looks to future work.

1.3 Exegetical basis

The book draws primarily on the *Critique of Pure Reason* and the so-called *Jäsche Logic*, which I will henceforth refer to as the *Critique* and the *Logic* respectively. I have said that the aim of the book is to understand the details of Kant's specific logical commitments as well as how these relate to certain of Kant's more general or high-level philosophical commitments. Generally speaking, the *Logic* is the basis for the former, the *Critique* for the latter. I hope that it is clear from the preceding that I take myself to have something substantial to say about the latter. Nevertheless, one of the main contributions of the book is its account of the former. I know of no other work that covers Kant's specific logical commitments in the detail or manner I do here. And for this, regarding both content and structure, I have largely followed the *Logic*—so much so that parts of the book can well be treated as a close reading or commentary on that work, specifically its “Doctrine of Elements”. Since Kant's *Logic* was not written by Kant, this requires comment.

The *Logic* was compiled and arranged by Gottlob Benjamin Jäsche at Kant's request, from Kant's heavily annotated copy of Georg Friedrich Meier's *Excerpt from the Doctrine of Reason*, which Kant took as the textbook for his own lectures on logic, as well as, in all probability, from one or more transcripts of those lectures. It was published in 1800 as *Immanuel Kant's Logic: a Manual for Lectures*. There is no evidence, however, that Kant himself took any role in preparing or reviewing the text, and there is no disputing that much of its presentation and organization is Jäsche's own or that significant aspects of its content are inherited from Meier. While it cannot be treated as a finished work written by Kant, it can nevertheless be treated as a “privileged transcript” of Kant's lectures on, and therefore commitments regarding, logic (see the discussion and citations in Young's Translator's Introduction, xvi–xix). To help allay any residual worries about its authority, I provide extensive cross-references to other logic transcripts from the Critical period. Finally, the doctrines that are really at the heart of my account are amply evidenced in the *Critique* as well as the in the *Logic*: Kant's specific account and systematic ordering of the possible forms of judgment; his insistence on the universal generality of concepts and the related principle that conceptual determination never terminates in a lowest species; and the primacy accorded to judgment as an essentially modalized activity of the understanding that first constitutes truth-evaluability. In light of this, I will occasionally allow myself to discuss passages of the *Logic* as though we could be sure that Kant himself wrote them, for instance by introducing a quotation with “Kant says...”. This is for purely stylistic reasons. I beg forgiveness from the sticklers.

Part I

General Logic

2 The Categorical Core

2.1 Introduction

We begin the reconstruction of Kant’s general logic with the smallest fragment needed to generate the traditional categorical forms and their relations of opposition. That means we admit two moments under **Quantity**:

Universal
Particular

Two moments under **Quality**:

Affirmative
Negative

And under **Relation**, we consider only:

Categorical

Crossing the two quantities with the two qualities under categorical relation yields the four traditional forms. Where *S* and *P* are our constituent concepts:

A : (All *S* are *P*) E : (No *S* are *P*)
I : (Some *S* are *P*) O : (Some *S* are not *P*)

The chapter proceeds in two stages. First we use Kant’s account of higher and lower concepts to define a strict partial order of conceptual determination, use this to derive the inverse relation between conceptual content and extension, and formulate the principle that there is no lowest concept (drawing primarily on the *Logic*, §§ 7–15, “Of concepts”). We then use Kant’s accounts of quantity and quality, together with the structure developed in the preceding section, to formalize the four categorical forms (drawing primarily on §§ 20–24, “Of judgments”).

It should be noted that much of the material we present in this chapter belongs to Kant’s inheritance from the rationalist tradition, and can accordingly be found in Meier’s *Auszug*, the textbook Kant took for his logic lectures (see e.g. §§249–67 on concepts and §§292–302 on judgments). The most important exception to this is Kant’s view that there is no lowest concept, which marks a decisive break with that tradition.¹ We will see in the next chapter how central this principle is to Kant’s logic, when we test our formulations of the four categorical forms against the square of opposition and conversion.

2.2 Concepts and the order of determination

Kant characterizes a concept as a representation through common marks (§ 1). Concepts are thereby general and mediate representations: they represent their objects through marks that themselves represent what can be shared by different objects.² He distinguishes what a concept contains *in* itself—its

¹The other main exception is Kant’s minimal account of negative quality as the mere denial of subordination, whereas Meier seems to require something like positive exclusion in speaking of “conflict” or “repugnance” (*Auszug* §§292–294). From Kant’s point of view this would elide the distinction between negative and infinite quality, so it will be important later when we turn to transcendental logic.

²See the Introduction §VIII, 9:58–65, for a full discussion of marks; cf. A68/B93, A320/B377.

content (*Inhalt*)—from what it contains *under* itself—its extension or sphere (*Umfang*). And he asserts, apparently as a matter of logic, that “the content and extension of a concept stand in inverse relation to one another. The more a concept contains *under* itself, namely, the less it contains *in* itself, and conversely” (§ 7).³ This is crucial as it motivates our first interpretative decision.

The content (*Inhalt*) of a concept consists of its marks, which are themselves concepts. Thus Kant also orders concepts as higher and lower, where a lower concept contains *in* itself the higher concept as a mark while adding further content of its own, that is, further marks not already contained in the content of the higher concept. Every concept will therefore be a lower concept relative to each of its marks, considered on their own (§ 9). Equivalently, every concept will be a species-concept relative to the genus-concepts defined by each of the marks it contains in itself (§ 10). This much is clear. But how we are to understand what falls *under* a concept, the constituents of its extension (*Umfang*)?

Kant’s texts permit two related notions of extension. A concept may be considered with respect to the lower concepts contained under it—its species and subspecies—or with respect to the objects that fall under it. Kant moves readily between these ways of speaking. For instance, the first time he introduces the content/extension distinction and their law of inverse relation (in §§ 7–8), Kant talks of ‘things’ falling under concepts to constitute their extension, while in the very next section, introducing the higher/lower distinction (§ 9), he talks of lower ‘concepts’ falling under higher ones to constitute their extension.⁴ Given the ease with which Kant moves between them, these descriptions should not be treated as rivals, but they do raise a question: Which notion of extension should be taken as primary in our initial reconstruction?

Kant’s claim about the inverse relation between content and extension gives us reason to begin with subordinate concepts as the constituents of the extensions of concepts. Kant presents the principle as a matter of logic: as a concept gains content, its extension narrows, and conversely. Yet this will not hold generally of the existing objects that happen to instantiate a concept. If every existing horse is white, then *white horse* would contain more marks than *horse* while exactly the same existing objects fall under both. Objectual extensions reflect contingent facts that need not register every difference of conceptual content.

Extension constituted by subordinate concepts makes the logical character of the principle intelligible. A further determination adds content to a concept, thereby producing a lower concept; the range of further concepts subordinate to it is correspondingly restricted relative to that of the higher concept. We will therefore begin with logical extension, understood in terms of possible further conceptual determinations. This gives the notion explanatory and expository priority without of course denying that concepts also possess a non-logical, objectual extension.⁵ Once the order of conceptual determination has been introduced, we will show formally that this interpretation validates a strict inverse relation between content and extension. Objectual extension will be introduced later, when we turn from conceptual determination to material truth and instantiation.

Focusing on logical extension, then, we can see that Kant describes the relation between higher and lower concepts in two equivalent ways. Considered in terms of content, the lower concept contains the higher concept as a mark while adding further marks of its own, as above. Considered in terms of logical extension, the lower concept falls under the higher. As Kant puts it in § 13: “The lower concept is not contained in the higher, for it contains more in itself than does the higher one; it is contained under it.”

³Cf. VL 24:911–12; DWL 24:753–56; A654–55/B682–83.

⁴The same shift is discernible throughout the *Critique*, including at key moments in its discussion of the theory of concepts, see e.g. A68–69/B93–95, A281/B337, A655–60/B683–88.

⁵In this I follow Anderson 2015 and Bader 2024. Lu-Adler 2012 proposes logical objects instead of concepts as the members of logical extension. Kant’s extremely broad use of terms like “object” is certainly permissive enough to make room for such a proposal, but it is explanatorily posterior and indeed reducible to the view developed here.

In § 15 he names the operation that produces this relation: “through continued logical determination ... lower and lower concepts arise.” This notion will serve as the primitive notion of our formalization.

2.2.1 Proper determination

Let C be a nonempty domain of admissible concepts, with $S, T, U, \dots$ as variables taking values in C . Unless otherwise indicated, all quantifiers in this chapter range over C . The letters have no objectual variables hidden inside them: they stand for concepts whose possible determinations will be compared.

We write

$$T \prec S$$

to mean that T is a *proper possible determination* of S . Thus T contains the content of S while adding some further admissible determination, and for that reason T is a lower concept, or species, relative to S . For example,

$$\text{rational animal} \prec \text{animal}.$$

Determination is *proper* in the sense that it is irreflexive:

Principle 2.1 (Irreflexivity).

$$\neg(S \prec S).$$

A concept does not count as a determination of itself. Kant never formulates this principle explicitly, but it is implicit throughout his discussion of the conceptual hierarchy—he is clearly thinking of determination as marking a genuine descent, from a higher concept to a *distinct* lower concept.⁶

Determination is also transitive:

Principle 2.2 (Transitivity).

$$U \prec T \wedge T \prec S \implies U \prec S.$$

If U determines T , and T determines S , then U determines S . Equivalently, whatever falls under a lower concept also falls under each concept higher than it. For example, suppose:

$$\text{rational animal} \prec \text{animal}$$

and:

$$\text{rational bipedal animal} \prec \text{rational animal}.$$

Then:

$$\text{rational bipedal animal} \prec \text{animal}.$$

This is the structural core of Kant’s rule that what belongs to a higher concept belongs also to the concepts subordinated under it (§14).⁷

Irreflexivity and Transitivity make $\prec$ a strict partial order of conceptual determination. No further structure is assumed at this stage. In particular, we do not yet assume that every concept S has a complementary concept *non-S*. Such resources will be introduced later as necessary.

⁶See especially §13, §15; cf. A68–69/B93–94; A655–60/B683–88.

⁷Cf. §63; A281/B337.

2.2.2 Logical extension and the Principle of Inverse Relation

Kant calls a concept higher insofar as it has lower concepts under it (§ 9), and broader insofar as its sphere, i.e. extension, contains those of its lower concepts (§ 12). We can represent the corresponding order of logical spheres by assigning to each concept the range of its proper possible determinations.

Definition 2.3 (Logical extension). *The logical extension, or determination sphere, of S is*

$$L(S) := \{T \in C : T \prec S\}.$$

The elements of $L(S)$ are concepts, not objects. Specifically they are the proper determinations of S , or the admissible ways in which S may be further specified. The set-notation provides an extensional representation of the determination order. It should not be taken to imply that the logical sphere is fundamentally a completed set rather than, for example, an open, branching structure of admissible further determinations. The set representation will be useful later for making the semantic basis of certain inferences perspicuous, but at this stage we leave its proper interpretation open.

This definition, together with Transitivity and Irreflexivity, yields Kant's view of the inverse relation between content and extension. Suppose that

$$T \prec S.$$

By Transitivity, every proper determination of T is then also a proper determination of S . Hence, by the definition of logical extension

$$L(T) \subseteq L(S).$$

Irreflexivity ensures this inclusion is proper: since $T \prec S$,

$$T \in L(S),$$

whereas Irreflexivity gives

$$T \notin L(T).$$

We therefore obtain:

Proposition 2.4 (The Principle of Inverse Relation).

$$T \prec S \implies L(T) \subsetneq L(S).$$

Every genuine increase in conceptual content thus produces a strictly narrower logical extension. Equivalently, the lower concept contains more *in* itself while the higher contains more *under* itself. The result follows from the order of conceptual determination alone; it does not depend on which existing objects happen to instantiate either concept.

2.2.3 The No-Bottom Principle

In § 11 Kant introduces a “universal law” that will be crucial in deriving the square of opposition with conversion, yet unlike much of the preceding, it is distinctively Kantian, without precedence in the rationalist tradition. Kant says: “in the series of species and genera there is no lowest concept (*conceptus infimus*) or lowest species, under which no other would be contained ... [hence] ... *there is no species that should not be able in turn to be a genus.*” Section 15 restates the point dynamically: “logical

determination can never be regarded as completed.”⁸ In short: there is no bottom to possible conceptual determination.

This principle receives extensive discussion and elaboration in the *Critique*. In the Appendix to the Transcendental Dialectic, Kant presents specification as a logical demand of reason: since the possible extent of a concept’s division cannot be read off from the sphere of the genus itself, reason demands that “no species be regarded as in itself the lowest”: every species remains a common concept and must therefore admit further subordinate concepts or subspecies (A655–57/B683–845). Kant is careful, however, to distinguish this logical principle from the claim that nature contains an actually infinite variety of things. The former asserts only the indeterminacy of a logical sphere with respect to possible further division and directs the understanding to continue seeking lower concepts (A656–57/B684–85).

We now have the resources to formalize this principle:

Principle 2.5 (No-Bottom, NB).

$$\forall S \exists T (T \prec S).$$

It follows immediately that every concept has a nonempty logical extension:

Corollary 2.6 (Nonempty logical spheres).

$$\forall S L(S) \neq \emptyset.$$

This is not an existence claim about objects. In particular, it does not imply that something is S : $\exists x Sx$ (where this quantifier ranges over objects). It says only that any admissible concept remains capable of further conceptual determination. Nevertheless, its structural similarity to the existence claim will be central to recovering the full square of opposition with conversion in the next chapter. And again, although the formal presentation quantifies over a domain of concepts, No-Bottom need not be understood as claiming that an actually completed infinite hierarchy of conceptual determinations is given in advance. It may instead express the open-ended rule that any conceptual specification can in principle be continued. Kant’s own formulations in the *Logic* and the *Critique* support just such a dynamic reading. We return to this point when the initially static semantics is reinterpreted in terms of the exercise of rules.

We now have the basic structure required for our reconstruction: an order of proper conceptual determination, the logical extensions induced by that order, and the principle that every concept admits further determination. We can now turn from concepts to judgments, specifically Kant’s account of the quantity and quality and categorical judgments.

2.3 Quantity and quality: deriving the four categorical forms

Kant describes the quantity and quality of judgments in §§ 21–22 respectively of the *Logic*: “In the universal judgment, the sphere of one concept is wholly enclosed within the sphere of another; in the particular, a part of the former is enclosed under the sphere of the other”; “In the affirmative judgment the subject is thought under the sphere of a predicate, in the negative it is posited outside the sphere of the latter.” These descriptions presuppose the categorical form of relation. In a categorical judgment, the constituent concepts are related directly as subject and predicate: as Kant puts it in § 24, the subject and predicate provide the matter of the judgment, while the copula expresses their relation. The categorical act thus brings S and P together within a single judgment; quantity determines how much of the subject sphere is taken, while quality determines whether what is taken is subordinated to, or excluded from,

⁸Compare VL, AA 24:911–12, and DWL, AA 24:755.

the predicate sphere. Specifically, universal quantity takes the whole sphere of the subject concept; particular quantity takes only a part of it; affirmative quality places what is taken under the predicate sphere; negative quality places it outside that sphere.⁹

The four categorical forms related in the square of opposition arise by crossing the two moments of quantity with the two moments of quality under a categorical relation, thus:

	Affirmative quality	Negative quality
Universal quantity	take the whole logical sphere of S and place it within the sphere of P	take the whole logical sphere of S and place it outside the sphere of P
Particular quantity	take some part of the logical sphere of S and place it within the sphere of P	take some part of the logical sphere of S and place it outside the sphere of P

We can use the logical spheres induced by proper determination to represent these forms. The formulas we define are:

$$\varphi ::= A(S, P) \mid E(S, P) \mid I(S, P) \mid O(S, P).$$

For each form, we give two equivalent clauses: first in terms of the order of proper conceptual determination, and then in terms of the logical spheres induced by that order. The latter is an extensional representation of the former, not a competing semantics.

2.3.1 Universal affirmative: All S are P

In a universal affirmative categorical judgment, the logical sphere of the subject concept is “wholly enclosed within” the logical sphere of the predicate concept (§ 21):

Definition 2.7 (Logical A).

$$\models_L A(S, P) \iff \forall T (T \prec S \rightarrow T \prec P).$$

Equivalently,

$$\models_L A(S, P) \iff L(S) \subseteq L(P).$$

Every possible determination of S is also a determination of P ; equivalently, the logical extension of S is a subset of the logical extension of P . The clause makes universal quantity explicit by ranging over the whole logical sphere of the subject. For example, *All horses are animals* just in case every admissible further determination of *horse* is also a determination of *animal*. No existing horse is required.

There is a minor complication. At the level of the categorical act, Kant sometimes describes universal affirmation more directly as the subordination of the subject concept *itself* to the predicate (see §§21–22, quoted above). Because determination is proper, however, the equivalence between these descriptions is not automatic. That is, our formal clauses for universal affirmation may appear to subordinate only the concepts properly below S to P , leaving it unclear whether S itself has been brought under P at all. If $S \prec P$, Transitivity guarantees that every proper determination of S also determines P , but the converse does not follow from Irreflexivity and Transitivity alone, since S is not among its own proper determinations. To recover direct conceptual subordination from the quantified clause, we therefore require, for distinct concepts, a sphere-representation principle:

$$S \neq P \wedge \forall T (T \prec S \rightarrow T \prec P) \implies S \prec P.$$

⁹Cf. DWL 24:764–66; A71–72/B96–97.

Equivalently:

$$S \neq P \wedge L(S) \subseteq L(P) \implies S \prec P.$$

This principle says that the proper determinations of a concept adequately represent its position in the conceptual hierarchy. Our determination-order clause and its set-theoretic representation already capture Kant’s description of universal affirmation in terms of the inclusion of logical spheres. This principle ensures that they also capture Kant’s description of universal affirmation as the subordination of the subject itself to the predicate: for distinct concepts, S is subordinate to P if and only if every proper determination of S also determines P . And the restriction to distinct concepts, $S \neq P$, preserves the degenerate case ‘All S are S ’: $A(S, S)$ still holds trivially, but it does not represent a genuine act of conceptual determination because Irreflexivity excludes $S \prec S$.

2.3.2 Particular affirmative: Some S are P

In a particular affirmative categorical judgment, “a part” of the logical sphere of the subject concept is enclosed under the logical sphere of the predicate concept (§ 21). On the present interpretation of logical spheres, this requires at least one possible determination falling under both the subject and the predicate concepts. We therefore need to introduce the following notion of compatibility:

Definition 2.8 (Compatibility).

$$S \not\prec P \iff \exists T (T \prec S \wedge T \prec P).$$

Two concepts are compatible in this sense when they admit at least one common possible determination. For example, *animal* and *white* are compatible if some admissible concept, such as *white horse*, is a determination of each.

Particular affirmation can now be represented as the compatibility of its subject and predicate concepts:

Definition 2.9 (Logical I).

$$\models_L I(S, P) \iff \exists T (T \prec S \wedge T \prec P).$$

Equivalently,

$$\models_L I(S, P) \iff L(S) \cap L(P) \neq \emptyset.$$

Some possible determination of S is also a determination of P ; equivalently, the logical extensions of S and P have at least one member in common—their intersection is nonempty. The common member is a possible conceptual determination. This does not say that an existing object is both S and P .

2.3.3 Universal negative: No S are P

In a negative judgment, Kant says, the subject sphere is “posited outside” the predicate sphere (§ 22). Combined with universal quantity, this requires the whole logical sphere of S to be outside that of P :

Definition 2.10 (Logical E).

$$\models_L E(S, P) \iff \forall T (T \prec S \rightarrow \neg(T \prec P)).$$

Equivalently,

$$\models_L E(S, P) \iff L(S) \cap L(P) = \emptyset.$$

No possible determination of S is also a determination of P ; equivalently, the logical extensions of S and P have no member in common—their intersection is empty. For example, *No squares are circles* just in case no admissible determination of *square* is also a determination of *circle*. Its strength comes from universal quantity: it is stronger than the bare claim that S itself is not subordinate to P . Note however that it does not yet require a primitive relation of exclusion, a definition of positive incompatibility, or a complementary concept *non-P*. Those stronger structures belong to the later treatment of infinite and disjunctive judgment.

As with A, we can distinguish the categorical act from its two equivalent formal representations. At the level of the act, the predicate P is denied of the subject S universally. Where affirmative quality involves subordination, negative quality involves the sheer denial of subordination. The latter is what Kant sometimes over-describes as positing the subject outside the sphere of the predicate, sometimes more accurately in terms of mere exclusion; universal quantity extends the exclusion throughout the whole subject sphere. In terms of the determination order, then, no possible determination of S is subordinated to P ; in terms of logical extension, the spheres of S and P are disjoint.

2.3.4 Particular negative: Some S are not P

Particular negative judgment takes some part of the subject sphere and posits it outside the predicate sphere:

Definition 2.11 (Logical O).

$$\models_L O(S, P) \iff \exists T (T \prec S \wedge \neg(T \prec P)).$$

Equivalently,

$$\models_L O(S, P) \iff L(S) \setminus L(P) \neq \emptyset.$$

Some possible determination of S is not a determination of P ; equivalently, the logical extension of S contains at least one member that does not belong to the logical extension of P —their set difference is nonempty. For example, *Some animals are not rational* if some admissible determination of *animal* does not fall under *rational*.

This clause should not be confused with the stronger claim:

$$\exists T (T \prec S \wedge T \perp P).$$

The latter says that the selected determination cannot share any further refinement with P . Still stronger is the infinite judgment that places it under a complementary concept *non-P*. Neither addition belongs to simple negative quality. The minimal clause states exactly what the quantity-quality composition requires and makes O the exact quantificational contradictory of the sphere-based A:

$$\models_L O(S, P) \iff \not\models_L A(S, P).$$

2.3.5 The initial categorical semantics

The four clauses can now be summarized (where all quantifiers range over C):

Form	Determination-order clause	Logical-extension clause
A	$\forall T (T \prec S \rightarrow T \prec P)$	$L(S) \subseteq L(P)$
E	$\forall T (T \prec S \rightarrow \neg(T \prec P))$	$L(S) \cap L(P) = \emptyset$
I	$\exists T (T \prec S \wedge T \prec P)$	$L(S) \cap L(P) \neq \emptyset$
O	$\exists T (T \prec S \wedge \neg(T \prec P))$	$L(S) \setminus L(P) \neq \emptyset$

The conceptual-hierarchy and set-theoretic columns are two representations of the same clauses, not competing logics: the latter gives an extensional representation of the former. In both, the witnesses required by particular quantity are possible conceptual determinations rather than existing objects. Negative quality is represented minimally as denial of subordination, leaving the stronger notions of complementary determination and exhaustive division for their proper place later in the reconstruction.

Presented together like this, the co-ordinate structure of the four forms is already discernible. The next chapter will test these clauses rigorously by using them to derive the complete square of opposition with conversion, as well as the categorical syllogistic.

3 Categorical Inference

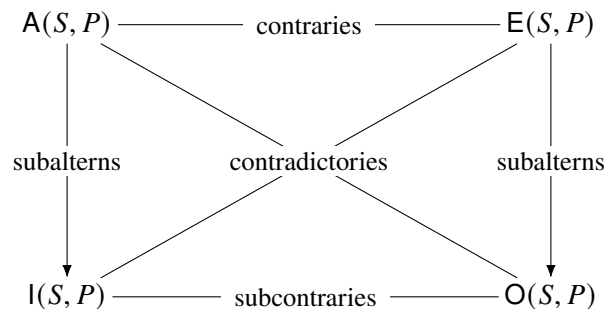
3.1 Introduction

The preceding chapter constructed the four categorical forms of judgment. We now turn from the construction of judgments out of concepts to the inferences licensed among judgments (drawing primarily on §§ 41–53 and §§ 56–70, “Of inferences”). Kant characterizes inference as “the derivation of one judgment from another” and distinguishes immediate inference, which derives one judgment from another “without a mediating judgment”, from mediate inference, which requires concepts other than those involved in the starting judgment, hence it requires a mediating judgment (§§ 41–42). The present chapter follows that progression. We begin with the immediate inferences, which Kant also calls “inferences of the understanding” (§ 43). These include all of the relations represented in the square of opposition (§§ 46–50), as well as the closely related conversion rules (§§ 51–53). The essence of such inferences consists “simply in an alteration of the *mere form* of judgments, while the *matter* of the judgments, the subject and predicate, remains *unaltered, the same*” (§ 44). The main result of this part of the chapter is to show precisely the work that Kant’s distinctive No-Bottom Principle does in validating certain immediate inferences—strikingly, it is required for exactly those traditional relations lost to modern logic. We then move on to categorical syllogisms, in which a middle concept mediates between the terms of the conclusion. The main result of this part of the chapter is to produce a natural explanation of Kant’s distinctive view of the privileged place of the first syllogistic figure.

Throughout the chapter we will sometimes speak of categorical formulas and inferences as logically correct or incorrect, even as true or false. This is provisional. Strictly speaking, the unmodalized formulas specify rules for possible syntheses and the relations among those rules; truth and falsity belong to exercises of such rules in acts of judgment proper. The distinction will be made explicit in the next chapter, when modality and the dynamic interpretation are introduced.

3.2 Immediate inference: the square of opposition and conversion

Recall the square of opposition:



Each of the four kinds of relations captured in the square is described in §§46–50 of the *Logic*: “In inferences of the understanding *per judicia subalternata* the two judgments are distinct as to *quantity*, and here the particular judgment is derived from the universal in consequence of the principle: The inference from the universal to the particular is valid” (§46); “two contradicting judgments cannot both be true, and just as little can they both be false. If the one is true, then the other is false, and conversely” (§48); “Contrary or conflicting judgments are judgments of which the one is universally affirmative, the other universally negative... In regard to these judgments... the inference from the truth of the one to the falsehood of the other holds, but not conversely” (§49); “Subcontrary judgments are ones of which the

one affirms or denies in particular what the other denies or affirms in particular. Since they can both be true but cannot both be false, only the following inference holds in regard to them: If one of these propositions is false, the other is true, but not conversely” (§50).¹⁰

The following sections of the *Logic* then go on to describe conversion rules that traditionally accompany the square itself and which invoke the same or similar inferential resources. Conversion consists in the “transposition of subject and predicate” (§ 51), i.e. switching S and P . A conversion is *simple* when quantity is preserved after transposition and *per accidens* when it is altered (§ 52). This rather general way of stating the matter yields eight possible further relations that we need to test—there are four categorical forms and two dimensions of variation. In practice, are only a few conversions are of interest. Aristotle discusses the four canonical cases in *Prior Analytics* I.2: universal affirmatives convert accidentally; universal negatives and particular affirmatives convert simply; particular negatives do not convert. That is:

$$\begin{aligned} A(S, P) &\longrightarrow I(P, S), \\ E(S, P) &\longrightarrow E(P, S), \\ I(S, P) &\longrightarrow I(P, S), \\ O(S, P) &\not\longrightarrow O(P, S). \end{aligned}$$

Of these, the *Logic* only explicitly covers the first three, valid cases (§ 53).¹¹ For completeness we will also cover the invalidity of O-conversion and briefly explain how verdicts on the other possible cases are either trivial or derivative on the canonical cases together with the resources already provided by the square.

As is his wont, Kant aligns the different immediate inferences of the understanding to different headings in his table of the logical functions of judgment: subalternation is assigned to quantity, as involving only a shift in quantity; contradictory, contrary, and subcontrary, as three kinds of opposition, are all assigned to quality, presumably because quality is where negation resides; and the conversion rules are assigned to relation, as involving the subject-predicate switch. These alignments all make some sense, and we will suggest below that they may even be significant. More tenuous is the alignment of a final, and historically particularly controversial conversion rule, contraposition, with modality (§ 54–55). Since this rule introduces *non*-concepts and so requires more structure than we have so far introduced, we postpone its treatment until later.

The proofs that follow aim to show not only that the immediate inferences of the square and conversion hold on the present semantics; they also aim to explain why they hold in relation to Kant’s specific commitments. For readers who don’t want to go through the details, which are elementary, the results are summarised and discussed together at the end of the section.

3.2.1 Contradiction

A and O.

Proposition 3.1 (A/O contradiction).

$$\models_L A(S, P) \iff \not\models_L O(S, P).$$

Equivalently, unfolding the two clauses:

$$\forall T(T \prec S \rightarrow T \prec P) \iff \neg \exists T(T \prec S \wedge \neg(T \prec P)).$$

¹⁰Cf. HL §§90–92; DWL 24:769–70.

¹¹Compare HL, 92, and DWL, AA 24:770–71.

Proof. Negating the universal clause for $A(S, P)$, quantifier duality then gives

$$\neg \forall T (T \prec S \rightarrow T \prec P) \iff \exists T \neg (T \prec S \rightarrow T \prec P).$$

Elementary propositional logic gives

$$\neg \forall T (T \prec S \rightarrow T \prec P) \iff \exists T (T \prec S \wedge \neg (T \prec P)).$$

And the formula on the right is precisely the clause for $O(S, P)$. □

This result does not require the No-Bottom Principle. It follows from the compositional relation between universal quantity and simple negative quality. O supplies exactly the counterexample that falsifies the universal inclusion asserted by A . This is an important consequence of retaining the minimal negative clause: had O required a positively incompatible branch, the negation of A would not by itself have been sufficient.

E and I.

Proposition 3.2 (E/I contradiction).

$$\models_L E(S, P) \iff \not\models_L I(S, P).$$

Equivalently, unfolding the two clauses:

$$\forall T (T \prec S \rightarrow \neg (T \prec P)) \iff \neg \exists T (T \prec S \wedge T \prec P).$$

Proof. By quantifier duality and elementary propositional logic again, the clause for E says that no determination of S is a determination of P :

$$\neg \exists T (T \prec S \wedge T \prec P).$$

This is exactly the negation of the existential clause for I . □

Again, no structural principle beyond the clauses is required. I asserts a common determination; E denies that any such determination is available.

The two diagonal contradictions exhibit the same quantificational pattern in different ways. A/O oppose universal inclusion to a counterexample determination. E/I oppose universal exclusion to a common determination. In both cases, the contradiction belongs to the form of the clauses alone.

3.2.2 Subalternation and the work of No-Bottom

The vertical relations of the square do require a further principle. From a claim about every member of a sphere one can infer a claim about some member only if the sphere is nonempty. In our semantics the requisite nonemptiness is supplied by No-Bottom:

$$\forall S \exists T (T \prec S).$$

The witness is a possible subordinate determination, not an existing object.

Proposition 3.3 (A-to-I subalternation). *Assuming NB:*

$$\models_L A(S, P) \implies \models_L I(S, P).$$

Equivalently, unfolding the two clauses:

$$\forall T(T \prec S \rightarrow T \prec P) \implies \exists T(T \prec S \wedge T \prec P).$$

Proof. By NB, choose T such that $T \prec S$. Since $A(S, P)$, every determination of S is a determination of P . Hence $T \prec P$. The same T therefore witnesses $I(S, P)$. $\square$

Proposition 3.4 (E-to-O subalternation). *Assuming NB:*

$$\models_L E(S, P) \implies \models_L O(S, P).$$

Equivalently, unfolding the two clauses:

$$\forall T(T \prec S \rightarrow \neg(T \prec P)) \implies \exists T(T \prec S \wedge \neg(T \prec P)).$$

Proof. By NB, choose T such that $T \prec S$. Since $E(S, P)$, every determination of S is denied subordination to P . Hence $\neg(T \prec P)$. The same T therefore witnesses $O(S, P)$. $\square$

These proofs display the exact analogy with existential import while preserving the distinction between logical and material extension. Traditional subalternation requires that the subject term not be empty. Here the relevant sphere cannot be empty because every admissible concept remains further determinable. Nothing follows about whether any existing object instantiates S .

3.2.3 Contrariety

Proposition 3.5 (A/E contrariety). *Assuming NB:*

$$\neg(\models_L A(S, P) \wedge \models_L E(S, P)).$$

Equivalently, unfolding the two clauses:

$$\neg(\forall T(T \prec S \rightarrow T \prec P) \wedge \forall T(T \prec S \rightarrow \neg(T \prec P))).$$

Moreover, there are admissible determination structures satisfying NB in which both judgments fail.

Proof. By NB, choose $T \prec S$. If $A(S, P)$, then $T \prec P$. If $E(S, P)$, then $\neg(T \prec P)$. Hence the two universal judgments cannot both hold. Equivalently, the result can be derived from subalternation and contradiction: $A(S, P)$ entails $I(S, P)$, while $E(S, P)$ contradicts $I(S, P)$. Since subalternation depends upon NB, this alternative derivation makes explicit how contrariety inherits that dependence.

The two universal judgments can, however, both fail. Suppose the subject sphere contains at least two determinations T and U , with $T \prec P$ but $\neg(U \prec P)$. Then $A(S, P)$ fails because of U , while $E(S, P)$ fails because of T . Their simultaneous falsity is registered by the simultaneous correctness of the two subcontraries:

$$I(S, P) \wedge O(S, P).$$

$\square$

No-Bottom is essential to contrariety. If $L(S) = \emptyset$, both universal clauses would be vacuously satisfied: every determination of S would fall under P , and every determination of S would fail to fall under P , simply because there were no such determination of S . The No-Bottom Principle rules out exactly this collapse.

3.2.4 Subcontrariety

Proposition 3.6 (I/O subcontrariety). *Assuming NB:*

$$\models_L I(S, P) \vee \models_L O(S, P).$$

Equivalently, unfolding the two clauses:

$$\exists T(T \prec S \wedge T \prec P) \vee \exists T(T \prec S \wedge \neg(T \prec P)).$$

Moreover, there are admissible determination structures satisfying NB in which both judgments hold.

Proof. Suppose $I(S, P)$ fails. By E-I contradiction, $E(S, P)$ holds. By E-to-O subalternation, which uses NB, $O(S, P)$ holds. Thus the falsity of I implies the correctness of O . Equally, if O fails, A holds by A-O contradiction, and A-to-I subalternation, which uses NB, yields I . The two particular judgments cannot both fail.

They may, however, both be correct. This occurs whenever one determination of S falls under P and another does not:

$$\exists T(T \prec S \wedge T \prec P) \quad \wedge \quad \exists U(U \prec S \wedge \neg(U \prec P)).$$

There is no requirement that the witnesses T and U be the same. Indeed, as Kant points out in a note to the relevant principle (§50), the two judgments must be witnessed by different determinations, since one determination cannot both fall and fail to fall under P : the judgments can both hold *because* they can be witnessed by different determinations of S . $\square$

3.2.5 The complete square

Theorem 3.7 (The logical square). *Every determination structure satisfying irreflexivity, transitivity, and NB, with the categorical clauses defined in section 2.3, validates the complete traditional square:*

1. A and O are contradictories;
2. E and I are contradictories;
3. A implies I ;
4. E implies O ;
5. A and E are contraries;
6. I and O are subcontraries.

The dependency structure should be emphasized:

Relation	Basis
A/O contradiction	quantifier duality and propositional logic
E/I contradiction	quantifier duality and propositional logic
$A \Rightarrow I$ subalternation	NB, providing a determination of S
$E \Rightarrow O$ subalternation	NB, providing a determination of S
A/E contrariety	NB, ruling out simultaneous vacuous universals; equivalently, contradiction with subalternation, and so NB
I/O subcontrariety	contradiction with subalternation, and so NB

The result is already informative. The traditional square follows from the quantified categorical clauses, understood as ranging over conceptual determinations rather than existing objects, plus Kant's distinctive No-Bottom Principle alone. The categorical clauses themselves secure the contradictory relations; No-Bottom secures subalternation and so, through combining subalternation with contradiction, contrariety and subcontrariety as well.

Before further discussion let us turn to the conversion rules, which reveal a similar structure.

3.2.6 Conversion

Deriving the relations of the square itself was a key test of our semantics. Yet the *Logic* does not present a sharp, bipartite distinction between the relations of the square on the one hand and those of conversion on the other. The conversion rules are important for us because they test the relation between the directed act of categorical predication and the structural relations represented by our semantic clauses. To predicate P of S is not in general equivalent to predicating S of P . Some of our clauses are nevertheless symmetric: having a common determination and lacking any common determination are relations that hold in either direction. As Kant points out in §53, this is what explains the simple conversion of I and E. The inclusion relation represented by A, by contrast, is not symmetric, so its conversion will be accidental rather than simple and will require further resources, which again will be supplied by No-Bottom.

Simple conversion.

Proposition 3.8 (Simple conversion of I).

$$\models_L I(S, P) \iff \models_L I(P, S).$$

Equivalently, unfolding the two clauses:

$$\exists T(T \prec S \wedge T \prec P) \iff \exists T(T \prec P \wedge T \prec S).$$

Proof. A witness T such that $T \prec S \wedge T \prec P$ is also a witness to $T \prec P \wedge T \prec S$. □

Proposition 3.9 (Simple conversion of E).

$$\models_L E(S, P) \iff \models_L E(P, S).$$

Equivalently, in terms of common determinations:

$$\neg \exists T(T \prec S \wedge T \prec P) \iff \neg \exists T(T \prec P \wedge T \prec S).$$

Proof. $E(S, P)$ denies a common determination of S and P . The nonoccurrence of a common determination, like the occurrence of the same, is symmetric □

The symmetry belongs to overlap and non-overlap, i.e. the mereological features of logical spheres, not to the act of predication. The converted judgment is a new directed act licensed by a symmetric feature of the conceptual structure.

Conversion *per accidens*.

Proposition 3.10 (Conversion per accidens of A). *Assuming NB:*

$$\models_L A(S, P) \implies \models_L I(P, S).$$

Equivalently, unfolding the two clauses:

$$\forall T (T \prec S \rightarrow T \prec P) \implies \exists T (T \prec P \wedge T \prec S).$$

Proof. Choose $T \prec S$ by NB. From $A(S, P)$, infer $T \prec P$. Hence T is a common determination of P and S . $\square$

The same result can be factored through two of the immediate inferences already established: $A(S, P)$ entails $I(S, P)$ by subalternation, and $I(S, P)$ converts simply into $I(P, S)$. The weakening from universal to particular is essential. $A(P, S)$ does not in general follow from $A(S, P)$: inclusion of logical spheres is not symmetric. (Hence $A(S, P)$ does not convert simply, which is why Kant states the rule for A as “Universal affirmative judgments may be converted *only per accidens*” (§53, my emphasis).)¹²

Both derivations make visible why conversion *per accidens* depends upon No-Bottom. And the same general reasoning shows us that universal negative judgments also convert *per accidens*, although Kant does not state this as a separate rule:

Proposition 3.11 (Conversion per accidens of E). *Assuming NB:*

$$\models_L E(S, P) \implies \models_L O(P, S).$$

Equivalently, unfolding the two clauses:

$$\forall T (T \prec S \rightarrow \neg(T \prec P)) \implies \exists T (T \prec P \wedge \neg(T \prec S)).$$

Proof. By simple conversion, $E(S, P)$ entails $E(P, S)$. By E-to-O subalternation, the latter entails $O(P, S)$. $\square$

Note that the reasoning is the same as above, except that simple conversion is applied before subalternation. As in the accidental conversion of A, its dependence on subalternation means that it ultimately relies on NB.¹³

¹²Nor of course is the reverse quantity-altering inference valid, i.e. the strengthening of particular to universal quantity under affirmative quality: $I(S, P) \not\rightarrow A(P, S)$. A nonempty overlap between the logical spheres does not imply that $L(P) \subseteq L(S)$. Traditionally, conversion per accidens is defined so as to ignore this direction of quantity alteration, since it is obviously invalid in both cases (see the next footnote). We only include it here because of Kant’s unusually general definition, in §52, of conversion per accidens as simply involving the alteration of quantity, rather than specifically its weakening. The pre-Critical Blomberg Logic, which more closely follows Meier, is more traditional in this regard, talking of the “restriction” of quantity (BL 24:282).

¹³Here too, the weakening of quantity from universal to particular is essential (see the previous footnote). $E(P, S)$ does not in general follow from $O(S, P)$, for a nonempty difference between the logical spheres does not imply that those spheres are disjoint: a determination may lie in $L(S) \setminus L(P)$ while some other determination lies in $L(S) \cap L(P)$.

The limits of conversion.

Proposition 3.12 (O does not convert simply).

$$\models_L O(S, P) \not\Rightarrow \models_L O(P, S).$$

Equivalently, the following implication does not hold in general:

$$\exists T(T \prec S \wedge \neg(T \prec P)) \implies \exists T(T \prec P \wedge \neg(T \prec S)).$$

Proof. Let the nonempty spheres be represented by $L(S) = \{a, b\}$ and $L(P) = \{a\}$. Then $b \in L(S) \setminus L(P)$, so $O(S, P)$; but $L(P) \subseteq L(S)$, so $L(P) \setminus L(S) = \emptyset$, and $O(P, S)$ fails. $\square$

This completes our treatment of the conversion cases proper to general logic, valid and invalid:

Form	Quantity preserved / simple	Quantity altered / per accidens
A	$A(S, P) \leftrightarrow A(P, S)$	$A(S, P) \rightarrow I(P, S)$
E	$E(S, P) \rightarrow E(P, S)$	$E(S, P) \rightarrow O(P, S)$
I	$I(S, P) \rightarrow I(P, S)$	$I(S, P) \leftrightarrow A(P, S)$
O	$O(S, P) \leftrightarrow O(P, S)$	$O(S, P) \leftrightarrow E(P, S)$

These results all concern formal validity. As Kant notes, certain judgments may of course convert in special cases owing to the specific relations between their constituent concepts, but no such content-dependent inferences belong to general logic (§53).

Again, the dependency structure should be emphasized:

Conversion	Basis
Simple conversion of I and of E	symmetry of: common-determination / denial thereof
No simple conversion of A or of O	non-symmetry of: inclusion / difference between logical spheres
No conversion per accidens of I or of O	particular conditions do not entail universal ones
Conversion per accidens of A and of E	simple conversion and subalternation (of relevant forms), and so NB

Only the two valid conversions *per accidens* require the No-Bottom Principle, since each incorporates subalternation. All the remaining conversion results—whether of validity or invalidity—follow directly from structural features already expressed by the categorical clauses. In this bipartite division of semantic labour, the structure of conversion parallels that of the square itself.

3.2.7 Discussion: No-Bottom, no empty terms, no existence, no possibilia

Our construction has reproduced two familiar facts about the square of opposition and conversion. First, contradiction and subalternation are jointly sufficient but separately insufficient to generate the full square. Second, this reflects a more fundamental duality in the semantic basis of immediate inference generally, between the work done by the internal structure of the categorical clauses, on the one hand, and the work done by a principle external to the structure of the categorical clauses themselves, on the other.

Contradiction belongs to the first side of this division in the semantics. It follows from the structure of the categorical clauses themselves. Subalternation belongs to the second: it requires a further condition ensuring that the subject-term is nonempty. Contrariety and subcontrariety inherit both sources of validity, since they follow from contradiction and subalternation jointly. And the conversion results reproduce the same division. The valid simple conversions follow from symmetries internal to the relevant clauses, whereas the valid accidental conversions require subalternation and so depend on its further condition.

These are general points, and it is nice to reproduce to them. But the crucial point of all of this is that Kant's distinctive No-Bottom principle satisfies precisely the further condition required for subalternation and so the full square with conversion. By providing every concept with a further determination, No-Bottom ensures that every logical sphere is nonempty, supplying exactly what the clause-internal structure leaves open.

Seen in this light, the order of presentation in the *Logic* may be more revealing than it initially appears. Aristotle begins from contradiction and contrariety (*De Interpretatione* 6–7), while Meier presents contradiction, contrariety, and subcontrariety before stating subalternation, even though he employs subalternation in explaining the other relations (*Auszug*, §§342–45).¹⁴ Kant, by contrast, begins with subalternation (§46) before turning to contradiction and then the other relations (§§47–53), and he coordinates the sequence in the kinds of immediate inference with the moments of judgment (§45). Whatever role Jäsche may have played in the final organization of the text, this arrangement gives subalternation a more conspicuous and systematic position than it typically occupied.

There is evidence that Kant appreciated much of the dependency structure underlying this arrangement. The presentation in the *Logic* itself is compressed, but his handwritten notes explain contrariety and subcontrariety through their connections with contradiction and subalternation, and they explain conversion through the relations between conceptual spheres (16:696–701). What Kant admittedly does not do, in any passage I can find, is bring these strands together and say explicitly that No-Bottom ensures the nonemptiness of logical spheres and thereby validates subalternation. That final connection remains a result of our reconstruction. But it is not imposed upon an otherwise indifferent ordering of Kant's materials. The inference he places first, and associates specifically with quantity, is precisely the inference that requires witnesses, and so in our semantics No-Bottom; and once it is combined with internal structure of the categorical clauses, the remaining relations of the square and conversion follow.

And what further follows from this is equally striking as regards the relation between Kant's logic and modern logic: the immediate inferences that in Kant depend on his distinctive No-Bottom principle are exactly those that have no unrestricted counterpart in standard modern logic, where they require the additional assumption that the subject concept is instantiated by an existing object. No-Bottom thus identifies the structural source of a characteristic difference between traditional logic and standard modern systems, yet it does so entirely within Kant's account of the order of conceptual determination; that is, without existential import.

This is of broader historical interest in at least two ways. First, because of where it places Kant regarding standard accounts of traditional logic; second because of what it suggests about the relation between Kant and his rationalist predecessors.

As we saw in the Introduction, it can seem obvious to modern eyes that many of the relations of the square and conversion are not generally valid because they fail when the subject-term does not refer to an *existing* object. Accordingly, the standard modern view is that proponents of traditional logic, in committing to these inferences, must have assumed existential import for all terms and judgments. An alternative, more nuanced approach agrees that traditional logicians assumed existential import for

¹⁴The same is true of the *Port Royal Logic* (Part II Ch.4).

affirmative judgments but argues that they could and often did deny it for negative judgments, so long as they formulate O appropriately.¹⁵ The second proposal will be of interest later, but for now the relevant point is just that, on the present interpretation, Kant's version of traditional logic fits neither model. For Kant, the full square of opposition with conversion can be valid and yet entirely free of existential import. This is because of his logical conception of extension together with No-Bottom. Kant's logic does validate the square by ensuring that no terms are empty, but it does this by means of a structural condition on conceptual determination that says nothing of existence.

Finally, then, this may also be relevant to our understanding of Kant's rationalist predecessors. For the idea that conceptual extensions can be populated by concepts is one Kant certainly inherits from his rationalist predecessors (see e.g. Meier, *Auszug*, §§260–263). Plausibly, they would also accept something very much like the above interpretation of the four categorical forms (see e.g. Meier, §§249–67 and §§292–302, cited above). By contrast, Kant's insistence upon the indefinite extendibility of conceptual determination is entirely his own, and the rationalists would dispute it. Meier, for instance, allows the hierarchy to terminate in singular concepts, with a lowest species containing only such concepts beneath it (§§260–62). The question therefore arises how a rationalist framework without No-Bottom can secure the non-emptiness required by the square and conversion.

For judgments whose subjects are abstract concepts, Meier, for instance, already has an answer. Such concepts are abstracted from lower concepts, and even a lowest species contains singular concepts under it; their logical extension is not empty. The difficulty arises at the terminal level itself, if singular concepts are admitted as subjects of the same categorical forms. No further concept falls under a singular concept, and hence its extension is empty when extension is defined solely by strict conceptual determination. To provide the witness required by the full square and conversion, then, the rationalist could instead appeal to the individual represented by the singular concept. But since, presumably, such individuals may not exist—some singular concepts are not instantiated by existing objects—this would be to appeal to an ontology of *merely possible* objects.

Now, this proposal has some initial plausibility insofar as it is quite standard to attribute just such an ontology to Kant's rationalist predecessors, albeit not, as far as I know, in this specific connection—as a means to validate the square of opposition and conversion in the face of concerns about empty terms. But it requires further investigation and will ultimately depend on whether rationalist singular concepts (or complete individual concepts, etc.) are admitted as subjects of the same categorical forms as general concepts. Meier, for one, employs singular judgments in subalternation and the conversion rules that depend on it, thus in precisely the immediate inferences which require witnesses (§342, §347). I cannot explore this issue fully here. But a related, weaker point about Kant himself stands regardless and is salient to recent discussions in the literature: given his distinctive commitment to No-Bottom, and at least as far as general logic is concerned, Kant has no need for mere possibilia. I will argue later that this point generalises to Kant's views about the logic of material truth and existence.

Philosophically, one may query the viability of a semantics on which No-Bottom secures no empty terms yet with no existence and no possibilia. What are conceptual determinations such that they are always available? What is logic *about*? We turn to this problem in the next chapter. First let us turn briefly to mediate inference.

¹⁵The locus classicus of the standard view is Kneale and Kneale's landmark study, 1960, 60. For the alternative view see Wedin (1990) and Parsons 2025.